An Exact Description Of Far Off-Axis Conic Surfaces For Non-Rotationally Symmetric Surface Generation

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Extended Abstract

The ability to servo a tool in a coordinated fashion with a workpiece's rotation on a two-axis lathe permits the generation of non-rotationally symmetric surfaces. One application of this technology is the diamond machining of far off-axis segments for large mosaic mirrors. By generating an off-axis mirror segment in a non-rotationally symmetric fashion, diamond turning lathes need only be sized to mirror segments dimensions and not encompass the full axis-to-segment distance. This poster describes one aspect of this technology: the exact description of these far off-axis mirror surfaces in a fashion that is appropriate for this non-rotationally symmetric surface generation.

The most common sagitta equation used for the representation of aspheric optical surfaces is the following:

$$z = \frac{c\rho^2}{1 + \sqrt{1 - [k+1]c^2\rho^2}} + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 \cdots$$

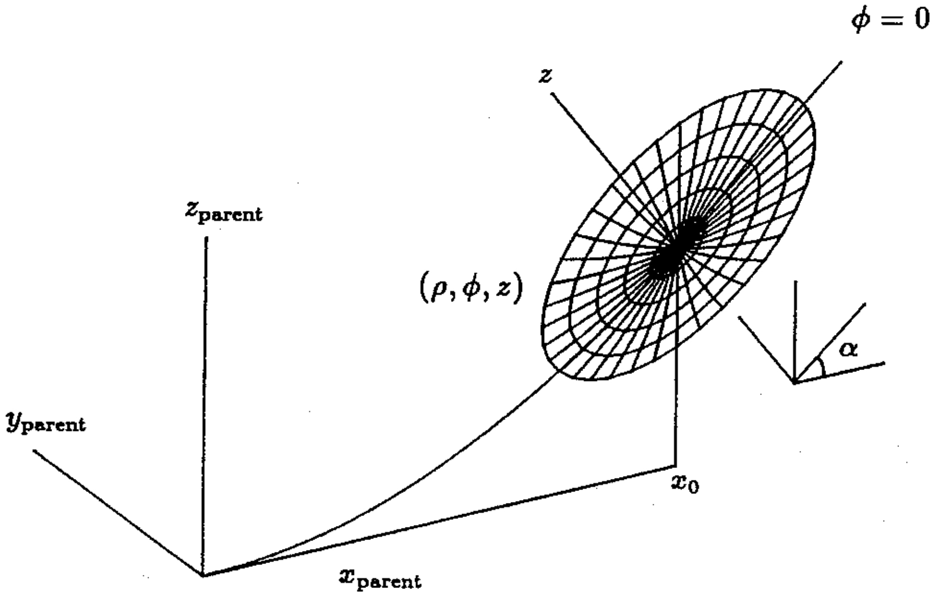
This equation defines a conic surface of revolution modified by simple polynomial terms in a cylindical coordinate system. In general the representation of a section of this surface for non-rotationally symmetric generation requires the solution of a transcendential equation. It is anticipated that most large mosaic mirrors will be unmodified conic surfaces of revolution.

Approximate representations of off-axis conic surfaces of revolution have been described in the literature for the fabrication of off-axis conic surfaces by deformation techniques.[1] An exact description for the case of a paraboloid with restrictions on orientation has also been reported by Thompson.[2] Following Thompson's derivation, the equation that exactly describes the geometry for a general far off-axis segment of a conic surface of revolution tilted at an arbitrary angle is the following:

$$z = d_1 + d_2\rho\cos(\phi) - \sqrt{d_3 + d_4\rho\cos(\phi) + d_5\rho^2 + d_6\rho^2\cos^2(\phi)}$$

In this equation z is the sagitta coordinate of the surface with respect to a coordinate system whose origin is located at a point on the parent conic with coordinates $(x_0, 0, z_0)$, and rotated

through an angle α. In this titled cylindrical coodinate system ρ is the radial coordinate and ϕ is the angular coordinate. The constants d_1 through d_6 are determine from the tilt angle α, the point $(x_0, 0, z_0)$, and the parent conic.



$$x_{\text{parent}}^2 = 2r z_{\text{parent}} - [k+1] z_{\text{parent}}^2$$

$$d_1 = \frac{x_0 \sin(\alpha) + r\cos(\alpha) + z_0 \sin(\alpha)}{1 + k\cos^2(\alpha)}$$

$$d_2 = \frac{-k\cos(\alpha)\sin(\alpha)}{1 + k\cos^2(\alpha)}$$

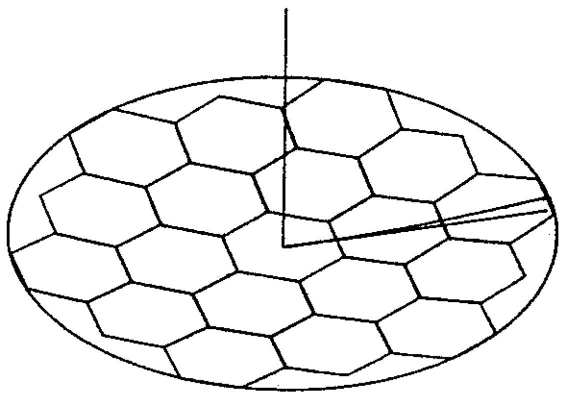
$$d_3 = d_1^2$$

$$d_4 = 2d_1 d_2 - \frac{2[x_0 \cos(\alpha) - r\sin(\alpha) + [k+1] z_0 \sin(\alpha)]}{1 + k\cos^2(\alpha)}$$

$$d_5 = \frac{-1}{1 + k\cos^2(\alpha)}$$

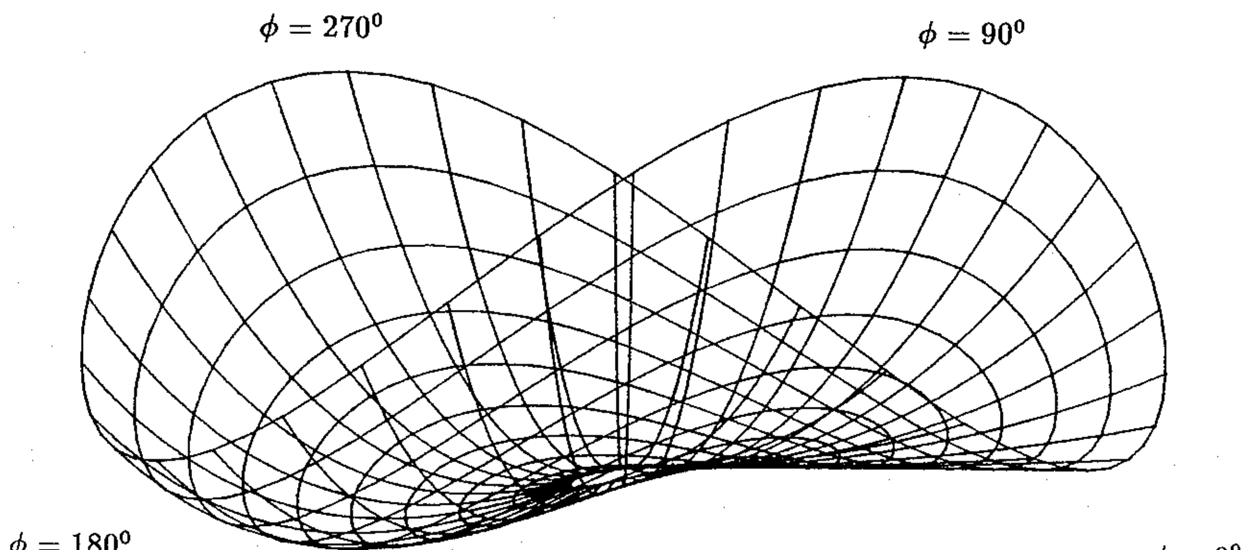
$$d_6 = d_2^2 - \frac{k sin^2(\alpha)}{1 + k\cos^2(\alpha)}$$

These equations provide a complete and exact description of the off-axis conic surface shape. Because rapid tool servo motions are restricted in displacement it is necessary to divide the surface geometry into two sets of coordinated machine motions. To generate the gross symmetric surface sagitta the two-axis orthogonal lathe motions are used. To generate the non-rotationally symmetric geometry the tool servo motion is used, coordinated with the workpiece rotation and slideway positions.

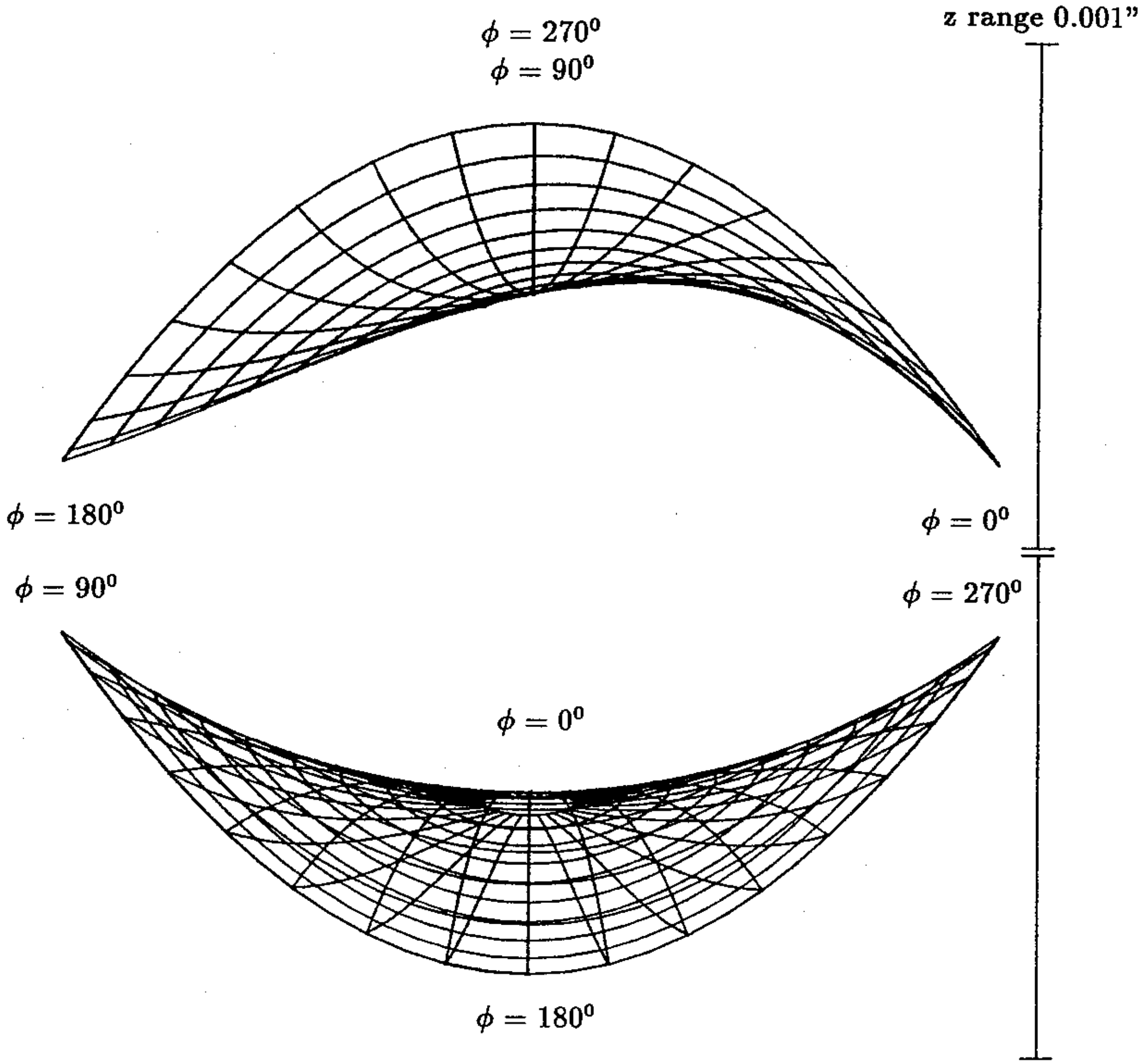


An example of this technology applied to a large segmented astronomical mirror is described below. In this example the parent surface is a paraboloid of revolution. The parent mirror functions at an optical speed of $f/2.5$, with a diameter of 80 inches and focal length of 200 inches. The mirror consist of 19 segments which are hexagonal as viewed along the optical axis. These segments are divided into 4 types: a center segment and three expanding rows of six segments each.

All 18 of these off-axis segments can be generated by this non-rotationally symmetric approach with a required fast tool servo motion of less than 0.001 inch! Two factors help to minimize the required servo tool motions. The first is an appropriate choice of α. Previous solutions for the parabolic case use the parent slope at the point $(x_0, 0, z_0)$, however, the choice of α that minimizes servo motion is that which at the maximum ρ yields the heights of the surface at $\phi = 0^0$ and $\phi = 180^0$ to be equal. The wire grid illustration below shows the servo tool motion required for the circumscribed outermost segment when a baseline motion is subtracted from each point on a given ρ equal to the height at $(z, \rho, \phi = 0)$.



The second factor influencing total servo motion is the subtraction of an optimum baseline. The optimum baseline is selected for each ρ position. Its value is the mean of the height extremes as ϕ progresses from 0^0 to 360^0. The illustrations below, for the same outside segment, show two views of the servo tool motion with this optimum baseline subtracted.



References

[1] Cardona-Nunez, Octavio *et. al.*, "Conic that best fits an off-axis conic section", Applied Optics, vol. 25, no. 19, 1986.

[2] Thompson, D.C., "Theoretical tool movement required to diamond turn an off-axis paraboloid on axis", Advances in the Precision Machining of Optics, SPIE Proc. vol. 93, 1976.